No. XVII.

On the Construction of Eclipses of the Sun. By John Gummere. Read March 20, 1829.

WE may, without diminishing the accuracy of the results, dispense with the description and division of an ellipse, which are necessary in the usual method of projecting eclipses of the sun, and which render it so troublesome. This is most conveniently done, by supposing the sun's centre to remain fixed in the centre of the circle of projection, and giving to the moon a parallax in right ascension, equal in magnitude, but opposite in direction, to the distance of the projection of the sun's centre from the universal meridian, at the time; and a parallax, parallel to the universal meridian, or parallax in declination*, equal in magnitude, but opposite in direction, to the distance of the sun's centre from a plane passing through the centres of the sun and earth, perpendicular to the universal meridian. The figure to which I shall refer, is the construction of an eclipse of the sun, that will occur on the 12th of February 1831. It is adapted to the meridian and latitude of Philadelphia.

The semicircle ACD represents the northern half of the circle of projection. AC is a parallel to the equator; SU is the universal meridian; SL a circle of latitude; PQ the

* These quantities are not, rigorously speaking, the moon's parallax in right ascension and declination, but it is convenient to call them so.

moon's relative orbit; the points 23, 0, 1, 2, and 3, on the line PQ, are the moon's places at the hours denoted by the numbers; the sun's place being supposed to be at S. All these are obtained as in the usual method.

Make the arcs AF and CH, each equal to the reduced latitude of the place, and join FH. With the centre N and radius NF, describe the quadrantal arc FU. Make FI and FK, each equal to the sun's declination, and join KI and FS. Draw *ar* and I*w*, parallel to FN and SD respectively. On NH, make NT equal to S*r*, and complete the rectangle NTXU. On XU, produced if necessary, take UV, equal to I*w*, taking it to the right of SU, when the sun's declination is north, but to the left, when the declination is south, and join VN. Take the hour angles from noon, corresponding to the hours marked on the relative orbit; and on the arc UF, produced if necessary, set off from U, arcs equal to these angles, marking their extremities with the numbers of the hours to which the arcs correspond. From the extremities of the arcs, draw lines parallel to UX or FH, as the lines 1, *x*; 2, *x*; &c. meeting NU in the points *u*, and NV in the points *v*. Then will the distances 1, *w*; 2, *w*; &c. be the moon's parallax in right ascension at the 23d, 1st, 2d, &c. hours; and the corresponding distances *vx*, will be the parallax in declination.

From the hour points on the moon's relative orbit, draw lines as 23, *n*; 1, *n*; &c. parallel to FH or AC; drawing them to the left hand, when the time is in the forenoon, but to the right hand, when the time is in the afternoon, and make them respectively equal to the parallax in right ascension at these hours. From the points *n*, draw the lines *n*, xxiii; *n*, O; &c. parallel to SU, drawing them below the point, *n*, and make them respectively equal to the moon's parallax in declination at the corresponding hours. Join xxiii, O; O, 1; &c. Then will the broken line thus formed be a near representation of the moon's apparent relative orbit; and the points xxiii, O, 1, &c. will be the moon's places in the apparent orbit at those times.

With the centre S, and sum of the semidiameters of the sun and moon, as a radius, describe arcs, cutting the apparent orbit in B and E, which will be the moon's apparent places at the times of beginning and end. From S, draw SG perpendicular to a straight line joining B and E; then G will be the moon's place at the time of greatest obscuration. And the point in which LS, produced if necessary, cuts the apparent orbit, is the moon's place at the time of apparent ecliptic conjunction. Take the distance between the hour point next preceding the point B, and that next following it; and applying it to the scale, obtain its measure. Do the same with the distance between B and the hour point next preceding. Then, as the 1st distance : 2d distance : : 60 minutes : the time past the preceding hour at which the eclipse begins. The other times are found in the same manner; and the quantity of the eclipse is found in the usual manner.

Find the moon's parallax in right ascension and declination for the time of beginning, and make S*z* equal to the parallax in declination. From *z*, draw *z*Z, parallel to FH, drawing it to the right hand when the time is in the forenoon, but to the left when it is in the afternoon, and make it equal to the parallax in right ascension. Join SZ, which will represent a vertical circle passing through the sun's centre; and the angle BSZ will be the angular distance from the sun's vertex, of the point at which the eclipse commences.

The slight changes necessary in the construction, for places near the equator or in the southern hemisphere, are so obvious as not to require notice.

In finding the times of beginning, &c. the moon's motion in the apparent orbit is assumed to be uniform during the hour, which is not strictly true. The greatest error, however, that can arise from the assumption, is only about a minute, when the latitude of the place is 40°. For higher latitudes it will be less; and for places nearer the equator it will be rather more. The error that may arise from assuming the part of the apparent orbit, between two consecutive hour points, to be a straight line, will seldom be as great as

that which is sometimes produced by omitting to diminish the sun's semidiameter; and this is usually omitted in the common method of projection. If the construction is made for each half hour, instead of each hour, which may be done with but little additional trouble, the error arising from the assumptions which have been mentioned will always fall within the unavoidable error of construction.

This method of construction is equally applicable to occultations of a star or planet.

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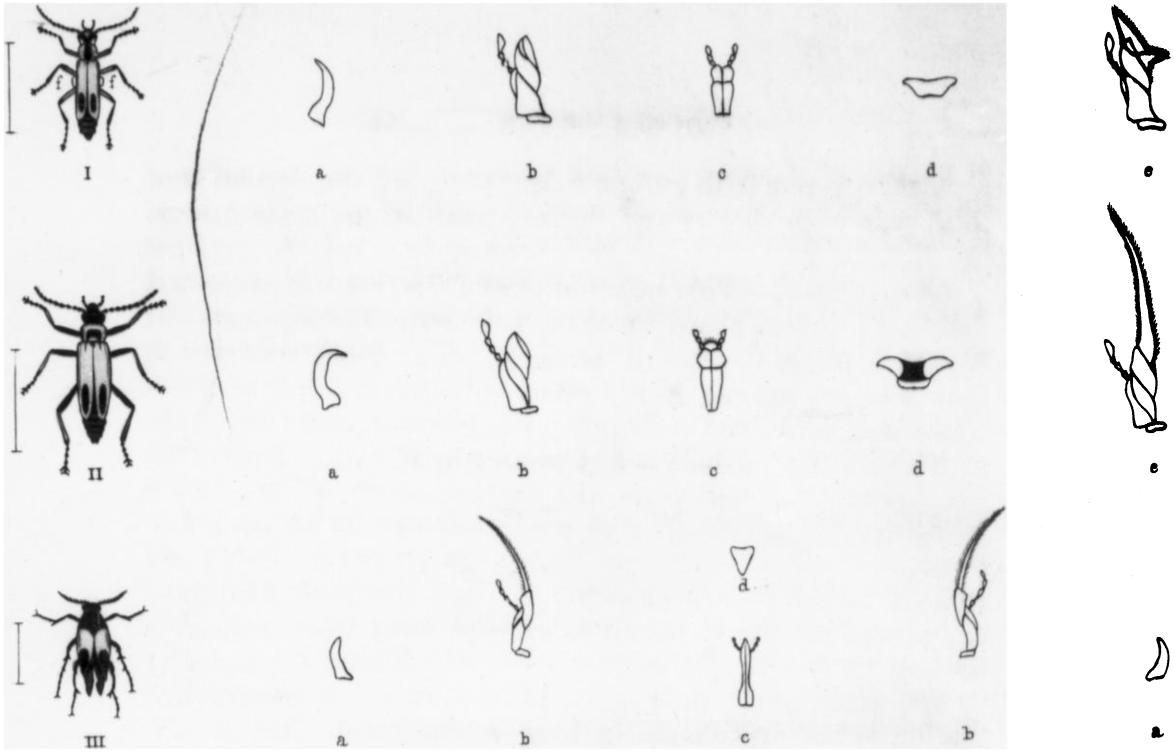

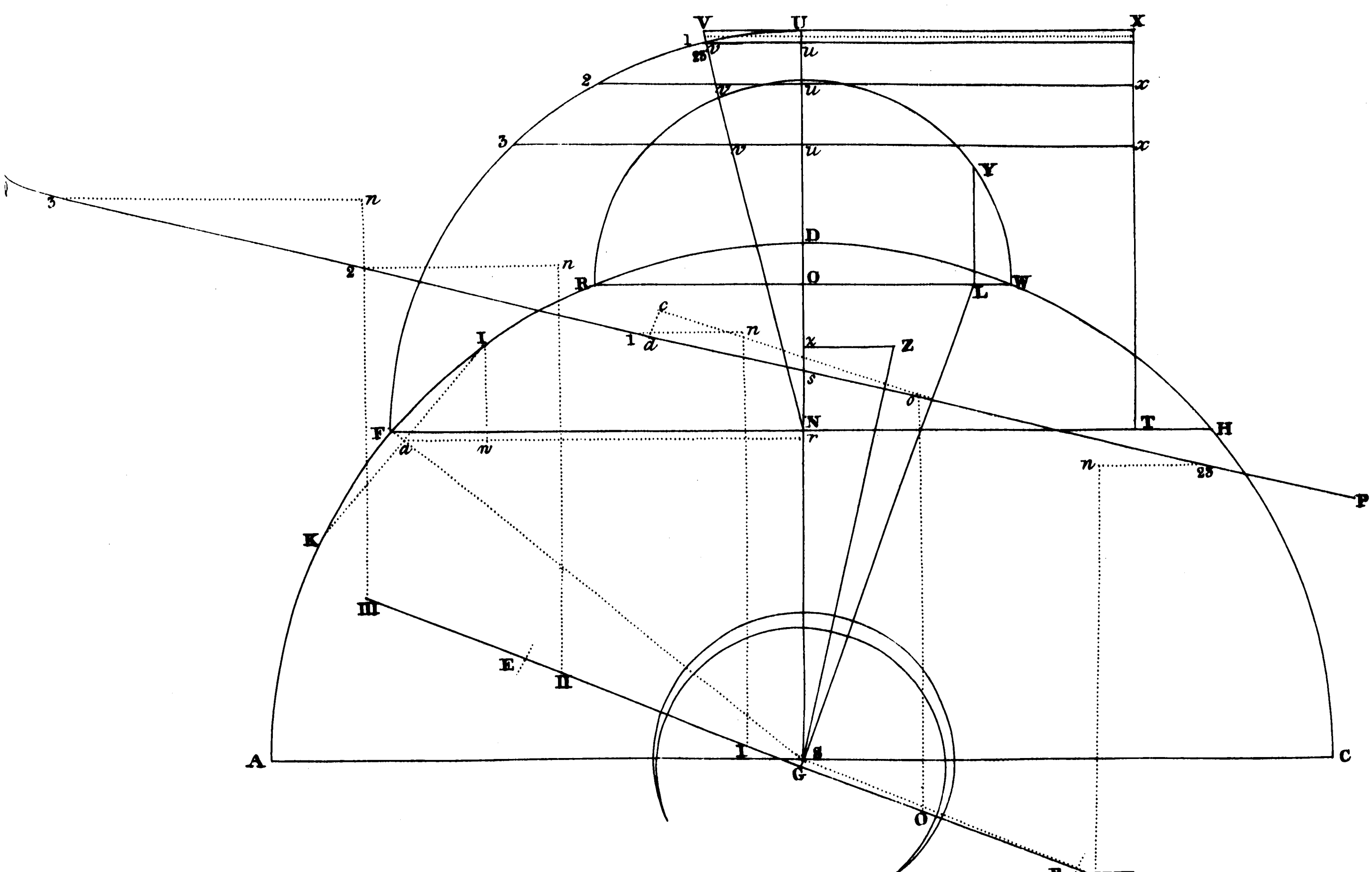

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